UNITED STATES DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
Washington, D. C. 20250

March 5, 1964

Mr. Orrin Ferris
709 South 7th
Bozeman, Montana

Dear Mr. Ferris

The graph you mention in your letter of March 1 must be our standard drawing ES-1001, which is included in the Soil Conservation Service "Hydrology Guide". The equation you quote should be considered separately from the soil-cover complex numbers. The equation is based on a hypothesis, but the numbers are empirically derived from research data.

As for the equation, it is only the most recent of a succession of runoff equations used by the SCS in the last twenty-five years. In the 1930's the infiltration theory was widely used and we had some very complicated procedures for estimating runoff on that basis. Two major difficulties in applying the infiltration theory were that (i) A large number of expensive infiltrometer runs had to be made to develop infiltration curves for an area, and (ii) Rainfall intensities were seldom available for areas we were interested in. We could not install recording rain gages and wait several years for records to accumulate. We turned to the use of rainfall totals, since data from standard non-recording gages are generally available, and where such gages did not exist we could make reasonably good estimates of rainfall totals by transposition of data. Various methods of using rainfall totals were developed, some methods using equations and others being graphical but with an equation implied. I will not describe these methods, but it is necessary to mention their existence because the present equation is a continuation of their basic ideas.

When we developed the present equation we had in mind the consolidation of various methods, practicality of application, availability of data, and so on. We therefore chose a simple hypothesis, which can be expressed as

$$\frac{F}{S} \text{ and } \frac{Q}{P} \rightarrow 1 \text{ as } P \rightarrow \infty$$

Spurious correlation

where F is the actual retention during a storm, S is the potential maximum retention, Q is the direct runoff (or the actual runoff), and P is the total storm rainfall (or the potential maximum runoff). So when considering flood producing storms we can express this in equation form as:

$$\frac{F}{S} = \frac{Q}{P} \qquad (1)$$

If we ignore the initial abstraction we can take:

$$F = P - Q \qquad (2)$$

and rewrite (1) as:

$$\frac{P - Q}{S} = \frac{Q}{P} \qquad (3)$$

By solving (3) for Q we get:

$$Q = \frac{P^2}{P+S} \qquad (4)$$

which is a runoff equation in which there is only one parameter, S, to which the storm and watershed characteristics are related. At this point it is necessary to say that (4) can be written as $Q = f(P, S)$, so that (3) is less simple than it appears. For example, the condition that $Q = P$ can occur only when S is zero, which does not happen under any natural circumstances. Even concrete surfaces have some retention.

In order to include the initial abstraction I_a we return to (2), which we write instead as:

$$F = (P - I_a) - Q \qquad (5)$$

so that (3) becomes:

$$\frac{(P - I_a) - Q}{S} = \frac{Q}{(P - I_a)} \qquad (6)$$

and by solving for Q we get:

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \qquad (7)$$

Equation (7) is a two-parameter equation, with both I_a and S related to storm and watershed characteristics. On the basis of the hypothesis, (7) is a true expression of a rainfall-runoff relation. But whether (7) is true in practical usage is another matter. We cannot test (7) until we know how to determine I_a and S. Of course we can plot curves for various assumed values of I_a and S and if this is done the curves have the typical appearance of accumulated runoff plotted versus accumulated rainfall.

It turns out, however, that I_a is a function of S. This is important, since (7) then becomes a 1-parameter equation and much easier to use. In order to find the relation between I_a and S we used rainfall and runoff data from research watersheds and plotted values of I_a versus their respective values of S, and got the empirical relation:

$$I_a = 0.2\ S \tag{8}$$

Using (8) in (7) gives us:

$$Q = \frac{(P - 0.2\ S)^2}{P + 0.8\ S} \tag{9}$$

which is the runoff equation used in ES-1001.

We now come to the question of the significance of S in terms of storm and watershed characteristics. We can relate S empirically to various characteristics, and if we do this properly we will have a systematic method for estimating runoff from rainfall. But which characteristics? Here again we drew on our experience. We chose to make S related to soil and cover conditions of a watershed. The storm characteristics could be included, but we preferred to have another way to account for them. In order to relate S to soil and cover characteristics of watersheds, we first developed the four categories of soils, then we listed the covers in which we were interested. We searched in the data literature for gaged watersheds having one soil type and one cover on them, of course finding many such watersheds but not for all combinations of soil and cover. We used what we did find. We plotted storm runoff versus storm rainfall for a particular watershed having one soil and one cover. We used one graph sheet per watershed. The plotted points scattered, but this was of no concern at the moment.

In order to find which value of S fit each watershed (and therefore the watershed's soil-cover complex), we first developed a system whereby the rainfall-runoff curves from (9) would plot at nearly equal intervals across a graph sheet. This was for easy interpolation, weighting of curve numbers, etc. The system consisted of substituting a runoff curve number (CN) for S, with the relation being:

$$CN = \frac{1000}{S + 10} \tag{10}$$

from which

$$S = \frac{1000}{CN} - 10 \tag{11}$$

By assuming values for CN we could find the appropriate values of S and use (9) to compute points for plotting rainfall-runoff curves. These curves are the ones shown in ES-1001.

By laying the rainfall-runoff sheet (ES-1001) over the data plotting for a watershed we could find that curve which divided the plotting into two equal numbers of points. The CN associated with this curve was therefore associated with the watershed soil-cover complex. By repeating this process with the watershed plottings we had we were able to find CN for most of the soil-cover complexes we had selected. The remaining CN were estimated by extrapolation or interpolation, using various plottings of the available CN. When we had several watersheds for a soil-cover complex the CN for that complex was an average of the CN for the individual watersheds. The CN associated with the soil-cover complexes are median values, roughly representing average conditions on a watershed. We took the average condition to mean average soil moisture condition, because we had to ignore rainfall intensity. Since watersheds can have above- and below-average moisture conditions, we again used the rainfall-runoff data plottings to find which CN fell near the upper envelope of the plotting and which near the lower. These envelope-CN were related to the average CN, and further study showed we could use 5-day antecedent rainfall as an index of whether to pick the upper, middle, or lower CN for a watershed. We have set up the method for the use of the average conditions, since there are many advantages to this, but the conversions to above- or below-average conditions are easy to make.

The last major step in the development was the testing of the runoff estimation method. Note that (9) or the set of derived CN cannot be tested alone. We made the customary comparisons of estimated runoff versus actual, and found the results satisfactory. There have been no changes in the method for the past nine years.

The above is a brief account of the development of the runoff equation and the soil-cover complex numbers. I have omitted many details. We did not accomplish the development overnight, nor did we exactly follow the sequence given above. With the statements in your letter in mind, I have the following comments:

1. The runoff equation is based on the hypothesis expressed by (1). "Justification" of the equation means the justification of (1). We justify (1) on the grounds that it produces rainfall-runoff curves of a type found on natural watersheds.

2. Other equations will also produce rainfall-runoff curves like those from (1) but these other equations have three or more parameters to be determined in advance, and this is difficult to do with ordinarily obtainable data.

3. Actually the CN's have been verified experimentally since they are based on data from research watersheds where the experiment was to determine the runoff for different soil and cover conditions.

4. The particular CN used by the SCS are not the only ones that can be developed for use with (9). By using other storm or watershed characteristics, other kinds of CN can be obtained. The practical value of the results will depend on how well the chosen characteristics can be represented by the data ordinarily at hand. We could have gone on to develop a very complicated set of CN, but they would have been unusable.

5. The research watersheds from which data were used are located in various parts of the United States, so that our CN apply throughout the country.

I hope you find this material suitable for your report. Your letter arrived late on the 4th and because you want a reply by the 9th I have had little time to write a polished account of our work. I am enclosing a copy of ES-1001 (in reduced size) in case the graph you have varies from the one I had in mind while writing this. The "G" on ES-1001 is the same as the "F" used in this letter.

With best wishes for the success of your report.

Sincerely yours,

Victor Mockus

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Enclosure

G on this sheet is the same as F in the letter.

HYDROLOGY: SOLUTION OF RUNOFF EQUATION $Q = \frac{(P-0.2S)^2}{P+0.8S}$

Q = 0 to 8 inches

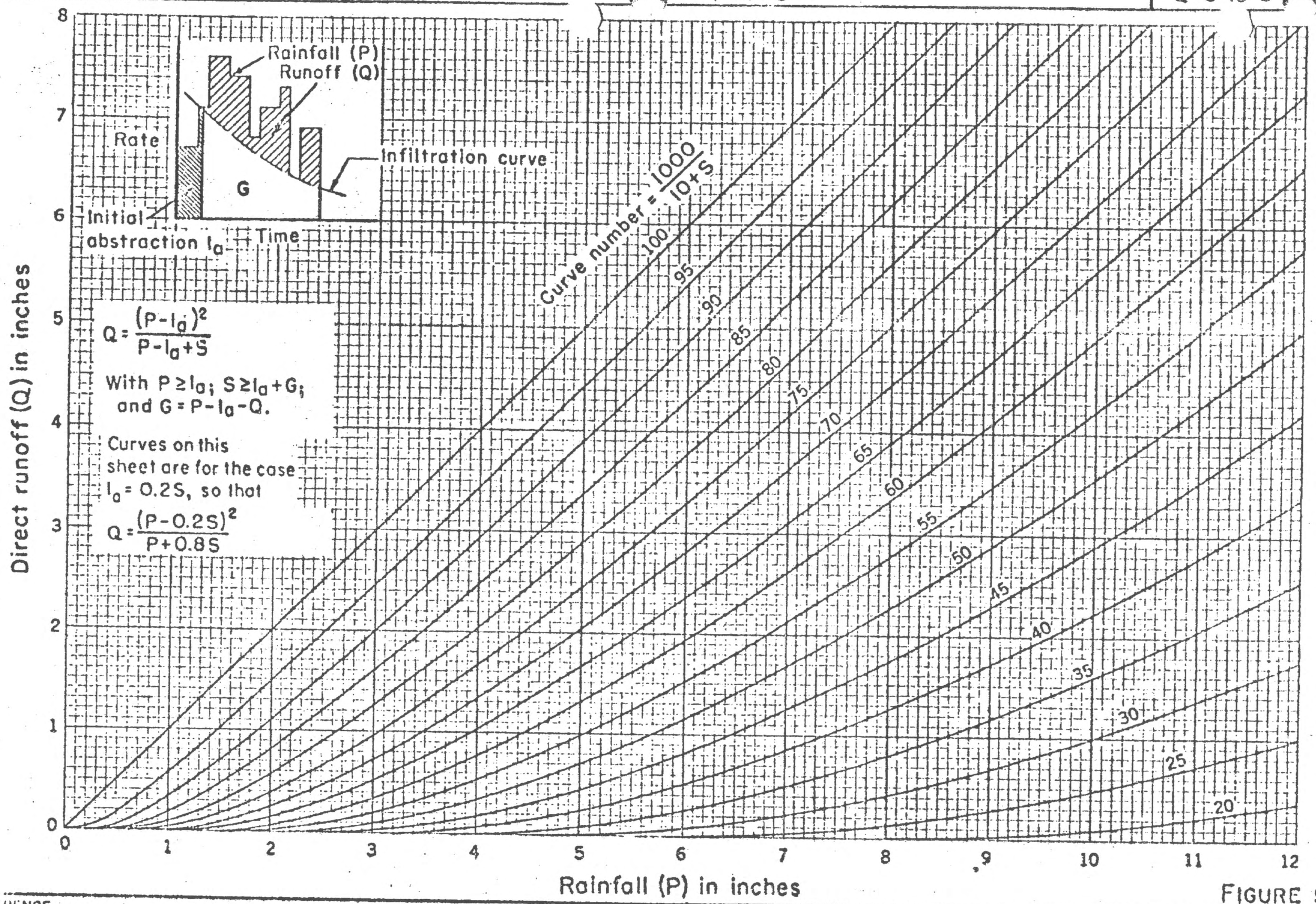

FIGURE 5.

REFERENCE

Mockus, Victor; Estimating direct runoff amounts from storm rainfall: Central Technical Unit, October 1955.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - CENTRAL TECHNICAL UNIT

STANDARD DWG. NO. ES-1001
SHEET 1 OF 2
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